PHYS302 Fall 2023

Homework 6

1. Using matrix ray tracing, find a single matrix that represents a thick lens with radii of curvature R1 and R2 and thickness d. Show that the final matrix reproduces equation 6.2 in your text (the equation for the focal length of a thick lens).

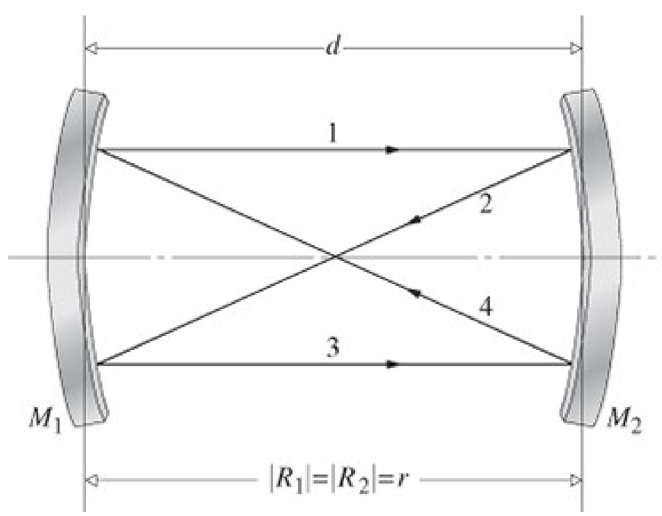
2. A positive meniscus lens with an index of refraction of 2.4 is immersed in a medium of index 1.9. The lens has an axial thickness of 9.6 mm and radii of curvature of 50.0 mm and 100 mm. Compute the system matrix when light is incident on the convex face.

3. Figure **P.6.29** (below) shows two identical concave spherical mirrors forming a so-called confocal cavity. Show, without first specifying the value of *d*, that after traversing the cavity two times the system matrix is

$$\begin{bmatrix} \left(\frac{2d}{r}-1\right)^2 - \frac{2d}{r} & \frac{4}{r}\left(\frac{d}{r}-1\right) \\ 2d\left(1-\frac{d}{r}\right) & 1-2\frac{d}{r} \end{bmatrix}$$

Then for the specific case of $d = r$ show that after four reflections the system is back where it started and the light will retrace its original path.

Figure P.6.29:



4. Starting with the exact expression given by Eq. (5.5), show that Eq. (6.46) results, rather than Eq. (5.8), when the approximations for l_o and l_i are improved a bit.

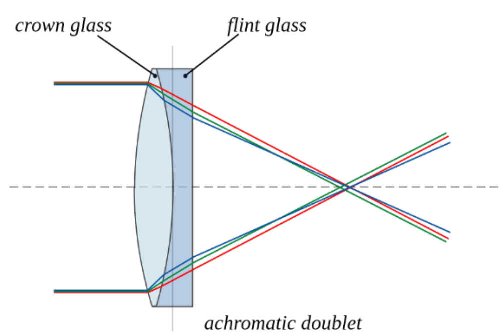
$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right) \tag{5.5}$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \tag{5.8}$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h^2\left[\frac{n_1}{2s_o}\left(\frac{1}{s_o} + \frac{1}{R}\right)^2 + \frac{n_2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)^2\right] \tag{6.46}$$

5. A thin achromatic doublet lens consists of two thin lenses in contact, forming a compound lens. A very common achromatic doublet consists of a positive lens made of crown glass (n_blue = 1.5231, n_red = 1.5138) affixed to a negative lens made of flint glass (n_blue = 1.6338, n_red = 1.6140).

Let the positive lens be convex-convex, and the negative lens be plano-concave, both as pictured below. The radii of curvature of 30.00 cm for all three curved interfaces (note: pay attention to sign of R, and recall that if the fourth interface is flat, R = infinity).



a. Calculate the focal length of the compound lens for red light and for blue light. How far apart will the two foci be?

b. Find the focal lengths for red and blue light if both lenses were made of crown glass. How much does the chromatic aberration increase? (Use f_blue - f_red to compare.)

Extra credit: (4 pts) A spherical mirror (a mirror with a surface that is described by the partial surface of a sphere) will create spherical aberrations in an image. A parabolic mirror (for which a cross section of the surface is described by a parabola) will fix this aberration.

For partial credit: explain why this is, using the definition of a parabola and of a circle, and draw a diagram as part of your explanation.

For full credit: (also) prove mathematically that the off-axis rays and paraxial rays are focused to the same point for the parabolic reflector but not the spherical one.